**Overview:** *The overview statement is intended to provide a summary of major themes in this unit.*

This unit builds on composition and decomposition of numbers that began in Prekindergarten with numbers up to ten and extends the work with numbers 11-19 in order to gain foundations for place value. It is important for students to become comfortable with whole numbers 11-19 and to understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine more. In Kindergarten, teachers help students lay the foundation for understanding the base-ten system by drawing special attention to 10 as a landmark number through hands-on activities and exploration. Prior to working with these teen numbers, Kindergarten students should have had many experiences decomposing 10 into pairs such as 1 and 9, 2 and 8, 3 and 7, and find the number that makes 10 when added to a given number such as 3. It is important that students’ place value work is intertwined with their work on counting and cardinality. It is also important to note that understanding our whole number base ten system is the foundation for later understanding of our decimal system.

**Teacher Notes:** *The information in this component provides additional insights which will help the educator in the planning process for the unit.*

* Review the Progressions for Grades K-5, Number and Operations in Base Ten at: <http://commoncoretools.files.wordpress.com/2011/04/ccss_progression_nbt_2011_04_073.pdf> to see the development of the understanding of number and operations as stated by the Common Core Standards Writing Team, which is also the guiding information for the PARCC Assessment development.
* When implementing this unit, be sure to incorporate the Enduring Understandings and Essential Questions as the foundation for your instruction.
* Since students at this age come to their development of base-ten concepts with a count-by-ones idea of number, teachers must begin there. In Kindergarten, the goal is to gain an initial foundation for place value. Students at this age should not be asked to explain that the 1 in 14 represents “one ten”. Their work with a group of ten and some ones leads to this understanding in Grade 1.
* By building the number concretely, students more easily make initial sense of foundations of the place-value system. In Kindergarten,
* students should use groupable base ten models, such as snap cubes or connecting cubes, versus pre-grouped base ten models, such as base

ten blocks. Groupable models most clearly reflect the relationships of ones and tens, for which the ten can actually be made or grouped

from ones.

* It is important to add estimation to grouping activities when working with place value so that students think about total quantities.
* Students must do more than regurgitate information. It is important that students construct the concept of place value rather than having the

concept of place value shown to or told to them. Like all mathematics concepts in the early grades, place value should be first taught as a

concept rather than as a procedure, and they should be using concrete materials to do so. Playing games that relate to real-life situations can

help children build their knowledge of place value and enrich their number sense.

* Using ten as a benchmark should be encouraged. When students see a set of four with a set of ten, they should begin to recognize that the total is 14 without counting. However, students at this age should not be expected to grasp the concept of a single ten.
* Teachers should strive to create a classroom environment in which students are encouraged to freely share their thinking about number and quantity.

**Enduring Understandings:** *Enduring understandings**go beyond discrete facts or skills. They focus on larger concepts, principles, or processes. They are transferable and apply to new situations within or beyond the subject.*

* There are many ways to represent a number.
  + - * Numbers can be composed and decomposed in a variety of ways.
      * Items can be grouped together to make them easier to count.
* Place value is based on groups of ten (10 ones = 10; 10 tens = 100).
* The digits in each place represent amounts of tens, or ones (e.g. 18 is 1 group of ten + 8 ones).
* There are patterns to the way numbers are formed. For example, in the teen numbers, the one remains fixed and the units change.

**Essential Questions:** *A question is essential when it stimulates multi-layered inquiry, provokes deep thought and lively discussion, requires students to consider alternatives and justify their reasoning, encourages re-thinking of big ideas, makes meaningful connections with prior learning, and provides students with opportunities to apply problem-solving skills to authentic situations.*

* How do I determine the most efficient way to represent a number (pictorial, symbolic, with objects) for a given situation?
* In what ways can numbers be composed and decomposed?
* In what ways can items be grouped together to make them easier to count?
* How does the position of a digit in a number affect its value?
* How are place value patterns repeated in numbers?
* How does using the base ten system make it easier for me to count?
* How does the place value system work?

**Content Emphasis by Cluster in Kindergarten:** *According to the Partnership for the Assessment of Readiness for College and Careers (PARCC), some clusters require greater emphasis than others. The table below shows PARCC’s relative emphasis for each cluster. Prioritization does not imply neglect or exclusion of material. Clear priorities are intended to ensure that the relative importance of content is properly attended to. Note that the prioritization is in terms of cluster headings.*

**Key:**

* ***Major Clusters***
* ***Supporting Clusters***
* ***Additional Clusters***

Counting and Cardinality

* Know number names and the count sequence
* Count to tell the number of objects.
* Compare quantities.

Operations and Algebraic Thinking

* Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

Number and Operations in Base Ten

* ***Work with numbers 11-19 to gain foundations for place value.***

Measurement and Data

* Describe and compare measurable attributes.
* Classify objects and count the number of objects in each category

Geometry

* Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).
* Analyze, compare, create, and compose shapes.

**Focus Standards:** *(Listed as Examples of Opportunities for In-Depth Focus in the PARCC Content Framework document):*

*According to the Partnership for the Assessment of Readiness for College and Careers (PARCC), this component highlights some individual standards that play an important role in the content of this unit. Educators should give the indicated mathematics an especially in-depth treatment, as measured for example by the number of days; the quality of classroom activities for exploration and reasoning; the amount of student practice; and the rigor of expectations for depth of understanding or mastery of skills.*

* **K.CC.B.4** Understand the relationship between numbers and quantities; connect counting to cardinality.
* **K.K.NBT.A.1** Work with numbers 11-19 to gain foundations for place value.

**Possible Student Outcomes:** *The following list provides outcomes that describe the knowledge and skills that students should understand and be able to do when the unit is completed. The outcomes are often components of more broadly-worded standards and sometimes address knowledge and skills necessarily related to the standards. The lists of outcomes are not exhaustive, and the outcomes should not supplant the standards themselves. Rather, they are designed to help teachers delve deeply into the standards and augment as necessary, providing added focus and clarity for lesson planning purposes. This list is not intended to imply any particular scope or sequence.*

The student will:

* Compose and decompose numbers from 11-19 into ten ones and some further ones (e.g. by using objects or drawings), and record each composition or decomposition by a drawing or equation (e.g. 18 = 10 + 8).
* Gain an understanding that the numbers 11-19 are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones. (Kindergarten students should see addition and subtraction equations, and student writing of equations in kindergarten is encouraged, but it is not required).
* Students explore and represent numbers 11-19 using representations, such as manipulatives or drawings. Using groupable models,

snap cubes, or connecting cubes allows students to clearly reflect the relationships of ones and tens, and hundreds for which the ten

can actually be made and grouped from ones. It is important that students construct the concept of place value rather than having the

concept of place value shown to or told to them.

**Progressions from Common Core State Standards in Mathematics:** *For an in-depth discussion of the overarching, “big picture” perspective on student learning of content related to this unit, see:*

The Common Core Standards Writing Team (07 April, 2011). Progressions for Grades K-5 Number and Operations in Base Ten, *accessed at* <http://commoncoretools.files.wordpress.com/2011/04/ccss_progression_nbt_2011_04_073.pdf>

**Vertical Alignment:** *Vertical curriculum alignment provides two pieces of information: (1) a description of prior learning that should support the learning of the concepts in this unit, and (2) a description of how the concepts studied in this unit will support the learning of additional mathematics.*

* **Key Advances from Previous Grades:** Students in Prekindergarten:
* Count verbally to 10 by ones.
* Identify written numerals 0-10.
* Understand the relationship between numbers and quantities to 5, then to 10; connect counting to cardinality.
* Represent a number (0-5, then to 10) by producing a set of objects with concrete materials, pictures, or numerals.
* For any given quantity from 0 to 5, use objects or drawings to find the quantity that must be added to make 5.
* Explore the relationship between ten ones and ten.
* **Additional Mathematics:**
* In grade 1, students move from exploring numbers 11-19 using representations, such as manipulatives or drawings and keeping each count as a single unit, in which they use 10 objects to represent “10” to creating a unit called a ten (unitizing) as indicated in the First Grade CCSS Standard 1.NBT.B.2a: 10 can be thought of as a bundle of ten ones — called a “ten.”
* In grade 1, students understand that the two digits of a two-digit number represent amounts of tens and ones.
* In grade 2, students understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones.
* In grade 2, students understand that 100 can be thought of as a bundle of ten tens called a “hundred.”
* In grade 2, students understand that the numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).
* In grade 2, students count within 1,000; skip-count by 5’s, 10’s, and 100’s.
* In grade 2, students read and write numbers to 1,000 using base-ten numerals, number names, and expanded forms.
* In grade 2, students compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.
* In grades 3 and 4, students use place value understanding and properties of operations to perform multi-digit arithmetic.
* In grade 4, students generalize place value understanding for multi-digit whole numbers.
* In grade 4, students extend their understanding of place value to apply to decimals and fractions that are renamed as decimals.
* In grades 5 and beyond, students perform operations with multi-digit whole numbers and with decimals to the hundredths.

**Possible Organization of Unit Standards:** *This table identifies additional grade-level standards within a given cluster that support the over-arching unit standards from within the same cluster. The table also provides instructional connections to grade-level standards from outside the cluster.*

|  |  |  |
| --- | --- | --- |
| **Over-Arching**  **Standards** | **Supporting Standards**  **within the Cluster** | **Instructional Connections outside the Cluster** |
| **K.NBT.A.1:** Compose and decompose numbers from 11-19 into tens ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., 18=10+8); understand that these numbers are composed of ten ones, and one, two, three, four, five, six, seven, eight, or nine ones. |  | **K.CC.A.1:** Count to 100 by ones and tens.  **K.CC.A.2** Count forward beginning from a given number within the known sequence (instead of having to begin at 1).  **K.CC.A.3**: Write numbers from 0-20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects).  **K.OA.A.3:** Decompose numbers less than or equal to 10 in pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).  **K.OA.A.4:** For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation. |

**Connections to the Standards for Mathematical Practice:** *This section provides examples of learning experiences for this unit that support the development of the proficiencies described in the Standards for Mathematical Practice. These proficiencies correspond to those developed through the Literacy Standards. The statements provided offer a few examples of connections between the Standards for Mathematical Practice and the Content Standards of this unit. The list is not exhaustive and will hopefully prompt further reflection and discussion.*

***In this unit, educators should consider implementing learning experiences which provide opportunities for students to:***

1. **Make sense of problems and persevere in solving them.**
   1. Determine what the problem is asking for; compose, **decompose**, make a ten.
   2. Use concrete or virtual models or pictures to help conceptualize and solve problems.
2. **Reason abstractly and quantitatively.**
   1. Recognize that a number represents a specific quantity.
   2. Create a representation of a problem to demonstrate the meanings of different quantities.
3. **Construct Viable Arguments and critique the reasoning of others.**
   1. Compare the concrete or virtual models used by others with yours.
   2. Explain your thinking to others and respond to others’ thinking.
4. **Model with Mathematics**
   1. Construct visual models using concrete or virtual manipulatives, pictures, or drawings to represent problems in multiple ways.
5. **Use appropriate tools strategically**
   1. Consider the use of groupable models, using materials such as interlocking cubes, as appropriate.
6. **Attend to precision**
   1. Use mathematics vocabulary such as “a group of ten” when communicating ideas with others.
   2. Correctly compose and **decompose** numbers to 19.
   3. Correctly record each composition or **decomposition** by using a drawing or models.
7. **Look for and make use of structure.**
8. Make observations about the patterns in teen numbers.
9. **Look for and express regularity in reasoning**
10. Look for repetitive actions in counting and computation, such as all teen numbers except eleven and twelve end with the word ‘teen’.

**Content Standards with Essential Skills and Knowledge Statements and Clarifications:** *The Content Standards and Essential Skills and Knowledge statements shown in this section come directly from the Maryland State Common Core Curriculum Frameworks. Clarifications were added as needed. Educators should be cautioned against perceiving this as a checklist. All information added is intended to help the reader gain a better understanding of the standards.*

| **Standard** | **Essential Skills and Knowledge** | **Clarification** |
| --- | --- | --- |
| **K.NBT.A.1**  Compose and decompose numbers from 11-19 into tens and ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., 18=10+8); understand that these numbers are composed of ten ones, and one, two, three, four, five, six, seven, eight, or nine ones. | **Essential Skills and Knowledge**   * **Ability to use concrete groupable materials (e.g., connecting cubes, snap cubes, etc.) to represent the combination of one ten and ones for each number** * **Ability to record the representations of 11 through 19 in pictures, numbers, and/or equations** | Special attention needs to be paid to this set of numbers as they do not follow a consistent pattern in the verbal counting sequence.   * Eleven and twelve are special number words. * “Teen” means one “ten” plus ones. * The verbal counting sequence for teen numbers is backwards – we say the ones digit before the tens digit. For example “26” reads tens to ones (twenty-six), but 16 reads ones to tens (six - teen). * To develop a strong sense of place value, students should have practice both reading the number as well as describing the quantity. For example, for 19, the students should read “nineteen” and state that it is one group of ten *and* nine more. Some students may be able to record that 19 = 10 + 9. Teaching the teen numbers as one group of ten and some extra ones is foundational to understanding both the concept and the symbol that represent each teen number. For example, when focusing on the number “14,” students should count out fourteen objects using one-to-one correspondence and then use those objects to make one group of ten ones and four additional ones. Students should connect the representation to the symbol “14.” Students should recognize the pattern that exists in the teen numbers; every teen number (except eleven and twelve) is written with a 1 (representing one ten) and ends with the digit that is first stated. * Using groupable models such as interlocking cubes allows students to clearly reflect the relationships of ones and tens, for which the ten can actually be made and grouped from ones. It is important that students construct the concept of place value rather than having the concept of place value shown to or told to them. It is unnecessary to use pre-grouped materials, such as base ten blocks, with students in Kindergarten. * When using snap cubes or connecting cubes, monochromatic versus multi-colored are more effective for transfer. For example, when building a train of 10 cubes, the cubes used should all be the same color. Otherwise, the student can be distracted by the different colors used or the patterns within the train. * It is important for students to use both concrete groupable base ten materials and virtual manipulatives. * The use of a double ten frame allows students to build a set of ten and some more. When using the double ten frame vertically, students should fill the left-hand frame first. This is so that when writing the number represented, the student can see that the "1" lines up with the group of ten, and the extra ones line up with the corresponding digit. By using the ten frames vertically and filling the squares from the bottom, the student can visualize the number they are working with.     **Ten plus four more ones.**  **14 = 10 + 4**   * Students should also practice using the double ten frame horizontally, as well.     **Ten plus four more ones.**  **10 + 4 = 14**   * Children can use layered place value cards to see the 10 “hiding” inside any teen number. Such decompositions can be connected to numbers represented with objects and drawings. |

**Evidence of Student Learning:** *The Partnership for the Assessment of Readiness for College and Careers (PARCC) has awarded the Dana Center a grant to develop the information for this component. This information will be provided at a later date. The Dana Center, located at the University of Texas in Austin, encourages high academic standards in mathematics by working in partnership with local, state, and national education entities.  Educators at the Center collaborate with their partners to help school systems nurture students' intellectual passions.  The Center advocates for every student leaving school prepared for success in postsecondary education and in the contemporary workplace.*

* Will be added when information is available.

**Fluency Expectations and Examples of Culminating Standards:** *This section highlights individual standards that set expectations for fluency, or that otherwise represent culminating masteries. These standards highlight the need to provide sufficient supports and opportunities for practice to help students meet these expectations. Fluency is not meant to come at the expense of understanding, but is an outcome of a progression of learning and sufficient thoughtful practice. It is important to provide the conceptual building blocks that develop understanding in tandem with skill along the way to fluency; the roots of this conceptual understanding often extend one or more grades earlier in the standards than the grade when fluency is finally expected.*

* Add and subtract within 5 in Kindergarten.

**Common Misconceptions:** *This list includes general misunderstandings and issues that frequently hinder student mastery of concepts regarding the content of this unit.*

* Thinks of a two-digit number additively in terms of ones. Example: 13 is thought of as 1 + 3 rather than 10 + 3.
* Recording a number that has zero ones. Example: 1 ten +0 ones = 1 instead of 10.
* Inadequate part-part-total knowledge for the numbers 0 to 10.
* An inability to trust the count.
* Little or no sense of numbers beyond 10 (e.g., fourteen is 10 and 4 more).

**Interdisciplinary Connections:** *Interdisciplinary connections fall into a number of related categories:*

* *Literacy standards within the Maryland Common Core State Curriculum*
* *Science, Technology, Engineering, and Mathematics standards*
* *Instructional connections to mathematics that will be established by local school systems, and will reflect their specific grade-level coursework in other content areas, such as English language arts, reading, science, social studies, world languages, physical education, and fine arts, among others.* 
  + *Will be added when information is available.*

|  |  |  |
| --- | --- | --- |
| **Available Model Lesson Plan(s)** | | |
| The lesson plan(s) have been written with specific standards in mind.  Each model lesson plan is only a MODEL – one way the lesson could be developed.  We have NOT included any references to the timing associated with delivering this model.  Each teacher will need to make decisions related ot the timing of the lesson plan based on the learning needs of students in the class. The model lesson plans are designed to generate evidence of student understanding.  This chart indicates one or more lesson plans which have been developed for this unit. Lesson plans are being written and posted on the Curriculum Management System as they are completed. Please check back periodically for additional postings. | | |
| **Standards Addressed** | **Title** | **Description/Suggested Use** |
| K.NBT.A.1 | Work with numbers 11-19 to gain foundations for place value | Students use the activities to build the concept of teen numbers as ten and some more ones. |

|  |  |  |
| --- | --- | --- |
| **Available Lesson Seeds** | | |
| The lesson seed(s) have been written with specific standards in mind.  These suggested activity/activities are not intended to be prescriptive, exhaustive, or sequential; they simply demonstrate how specific content can be used to help students learn the skills described in the standards. Seeds are designed to give teachers ideas for developing their own activities in order to generate evidence of student understanding.  This chart indicates one or more lesson seeds which have been developed for this unit. Lesson seeds are being written and posted on the Curriculum Management System as they are completed. Please check back periodically for additional postings. | | |
| **Standards Addressed** | **Title** | **Description/Suggested Use** |
| K.NBT.A.1 | Using a Rekenrek to build and understand place Value | Students use a Rekenrek to build one ten and some ones. Then they record the number sentence that represents that number. They also work with word problems to build understanding of place value. |
|  |  |  |
|  |  |  |
|  |  |  |

**Sample Assessment Items:** *The items included in this component will be aligned to the standards in the unit and will include:*

* + *Items purchased from vendors*
  + *PARCC prototype items*
  + *PARCC public released items*
  + *Maryland Public release items*
  + *Formative Assessment*

|  |  |  |  |
| --- | --- | --- | --- |
| **Topic** | **Standards Addressed** | **Link** | **Notes** |
| Working with numbers 11-19 to gain foundation for place value under-standing | K.NBT.A.1 | <http://www.k-5mathteachingresources.com/support-files/mydoubletenframeriddle.pdf>  <http://www.k-5mathteachingresources.com/support-files/teens-on-the-rekenrek.pdf>    <http://www.k-5mathteachingresources.com/support-files/teens-on-ten-frame-book1.pdf>  <http://www.k-5mathteachingresources.com/support-files/tensandoneswithunifixcubes.pdf> | Students use a double ten frame to help them write riddles.  Students use a Rekernek  Students make ten frame books with the teen numbers  Students use interlocking cubes to build tens and ones  Please see the K-5 Math Teaching Resources site for Kindergarten at: <http://www.k-5mathteachingresources.com/kindergarten-math-activities.html>  Please see the Illustrative Mathematics site at <http://illustrativemathematics.org> for a variety of tasks to use with your students. |

**Interventions/Enrichments:** *(Standard-specific modules that focus on student interventions/enrichments and on professional development for teachers will be included later, as available from the vendor(s) producing the modules.)*

**Vocabulary/Terminology/Concepts:** *This section of the Unit Plan is divided into two parts. Part I contains vocabulary and terminology from standards that comprise the cluster which is the focus of this unit plan. Part II contains vocabulary and terminology from standards outside of the focus cluster. These “outside standards” provide important instructional connections to the focus cluster.*

***Part I – Focus Cluster:***

***decomposition:*** breaking a number into two or more parts to make it easier with which to work.

Example: When combining a set of 5 and a set of 8, a student might decompose 8 into a set of 3 and a set of 5, making it easier to see that the two sets of 5 make 10 and then there are 3 more for a total of 13.

Decompose the number 4; 4 = 1+3; 4 = 3+1; 4 = 2+2

***represent:*** display addition or subtraction processes using concrete materials, pictures, numbers, words, or acting it out.

***subitizing:*** the ability to recognize the total number of objects or shapes in a set without counting. Example: Recognizing that this face of a cube has five dots without counting them.

***Part II – Instructional Connections outside the Focus Cluster***

* ***one-to-one correspondence:*** *linking a single number name with one object--and only one--at a time.*
* ***cardinality:*** is the understanding that when counting a set, the last number represents the total number of objects in the set. Example:

( 1 2 3 )

This is a set of 3stars

* ***rote counting:*** reciting numbers in order from memory without aligning them to objects, pictures, etc.
* ***verbal counting:*** counting while aligning each number said to an object, picture, etc. in order to solve a problem.

**Resources:**

**Free Resources:**

* <http://wps.ablongman.com/ab_vandewalle_math_6/0,12312,3547876-,00.html> blackline masters, including ten frames and double ten frames
* <http://www.kentuckymathematics.org/resources/pimser.asp> blackline masters and games
* <http://teachmath.openschoolnetwork.ca/wordpress/kindergarten/number-sense/510-frames/> games, including the game Right On, in which students work to fill a ten frame, as well as ten frame activities.
* <http://mathwire.com/> mathematics teaching resources
* <http://www.nzmaths.co.nz/confusion-about-teen-numbers?parent_node> Ideas to help with confusion about teen numbers
* <http://sci.tamucc.edu/~eyoung/literature.html> links to mathematics-related children’s literature
* <http://reallyroper.blogspot.com/2011/01/numbers-in-teens-they-start-with-1-song.html?spref=bl> Video on teen numbers
* <http://exchange.smarttech.com/search.html?q=%22place%20value%22> Smart Board activities for place value
* <http://illuminations.nctm.org/ActivityDetail.aspx?ID=8> Electronic abacus tool
* <http://wps.prenhall.com/ca_ab_vandewalle_mathematic_1/25/6506/1665690.cw/-/1665698/index.html> Blackline Master with bean sticks
* <http://www.center.edu/BLACKLINES/001-061/033-039.pdf> Blackline Master with beans
* <http://digiblock.com/> free lesson plans
* <http://www.k-5mathteachingresources.com/support-files/tensandoneswithunifixcubes.pdf> K-5 Teaching Resources
* <http://illuminations.nctm.org/ActivityDetail.aspx?ID=218> NCTM Illuminations Game ‘Grouping and Grazing’
* <http://illuminations.nctm.org/LessonDetail.aspx?id=U153> Lesson Plans for building sets through 20
* <http://www.k-5mathteachingresources.com/support-files/mydoubletenframeriddle.pdf> “My Double Ten-Frame Riddle”

**Math Related Literature:**

* Martin, Bill Jr. 10 Little Caterpillars.

Notes: A different caterpillar appears on each spread and moves through a lovely garden scene. This final spread of this rhyming book features a countdown of each caterpillar.

* Wood, Jakki. One Tortoise, Ten Wallabies: A Wildlife Counting Book.

Notes: The counting units extend beyond 10.

**References:**

* ------.“The Developmental Bases for Early Childhood Number and Operations Standards. In *Engaging Young Children in Mathematics: Standards for Early Childhood Mathematics Education,* edited by Clements, D. H., Sarama, J., DiBiase, A.
* ------. 2000. *Principles and Standards for School Mathematics.* Reston, VA: National Council of Teachers of Mathematics.
* 2006. *Curriculum Focal Points for Prekindergarten Through Grade 8 Mathematics: A Quest for Coherence.* Reston, VA: National Council of Teachers of Mathematics.
* Arizona Department of Education. “Arizona Academic content Standards.” Web. 28 June 2010

<http://www.azed.gov/standards-practices/mathematics-standards/>

* Bamberger, H.J., Oberdorf, C., Schultz-Ferrell, K. (2010). *Math Misconceptions: From Misunderstanding to Deep Understanding.*
* Bamberger, H.J., Oberdorf, C. (2010). *Activities to Undo Math Misconceptions, PreK - Grade 2*. Portsmouth, NH: Heinemann.
* The Common Core Standards Writing Team. Progressions for the Common Core State Standards in Mathematics (draft). *K – 5, Number and Operations in Base Ten.* 7 April, 2011. Web. <http://commoncoretools.files.wordpress.com/2011/04/ccss_progression_nbt_2011_04_073.pdf>
* Copley, J. (2000). *The Young Child and Mathematics*. Washington, D.C.: National Association for the Education of Young People.
* North Carolina Department of Public Instruction. Web. February 2012. <http://www.ncpublicschools.org/docs/acre/standards/common-core-tools/unpacking/math/3rd.pdf>
* Ross, S. (1989). P*a*rts, wholes, and place value: A developmental view. *Arithmetic Teacher*, 36, 47-51.
* Shane, R. (1999). Making connections: A ‘number curriculum’ for preschoolers. In J.V. Copley (Ed.), *Mathematics*

*in the early years*, (pp. 129-134). Reston, VA: National Council of Teachers of Mathematics.

* Van de Walle, J. A., Lovin, J. H. (2006). *Teaching Student-Centered mathematics, Grades K-3.* Boston, MASS: Pearson Education, Inc.